UNIVERSITY OF COPENHAGEN

Data-Driven Financial Models

Analysis of Optimal Portfolios and Bonds

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1.1 Portfolio Choice

In choosing the portfolio, we focused on picking stocks from different sectors and markets. Specifically, the stocks are centered around American and German markets because of their constancy. Portfolios will then be made of stocks from the following companies: Walmart, The Coca-Cola Company, The Walt Disney Company, Nike, Allianz SE, Deutsche Lufthansa AG, Volkswagen and Adidas.
In the very beginning we thought about considering also Japanese stocks, owing to their position in the tech industry, however we discarded them in the end because we faced some problems when dealing with high numbers of stocks. What happened was that the result we obtained in the first data manipulations seemed very unrealistic (i.e many risk free stocks) and with few attempts in understanding what was going on, we presumed the problem came from the number of chosen stocks in relation with the length of the rolling window we analyzed later in the code that leads then to a singular covariance matrix.
So, moving further, we eventually selected, in a time interval from 1st January 1999 to 31st December 2018, 9 stocks representing a broad range of sectors such as food and beverage manufacturing, consumer discretionary, aviation, consumer services, entertainment and banking financial services.
The stock choices we made are based on the results of financial theory: diversification benefits are higher when stocks in a portfolio are uncorrelated or negatively correlated.
Generally, financial risk in a portfolio can be decomposed into *systematic* and *unsystematic* risk. Systematic risk is also referred to as undiversifiable risk and this component of risk we cannot mitigate. Systematic risk reflects major economic changes which affect the market as a whole. This could be changes in interest rate, recessions, inflation etc.
Contrary to this, unsystematic risk affects individual sectors or securities and can generally be avoided through proper diversification. This is done by constructing a portfolio in which stocks are uncorrelated or negatively correlated in order to hedge against the risk of a fall in a specific sector with the rise in another sector.
It is also worth noting that we restricted our financial data from 19 99 to 2019 in order to avoid market irregularities caused by COVID-19. In times of crisis we generally observe increased correlation between stocks in a portfolio and systematic risk tends to increase as the effects of the crisis materialize in the market.

1.2 Estimation

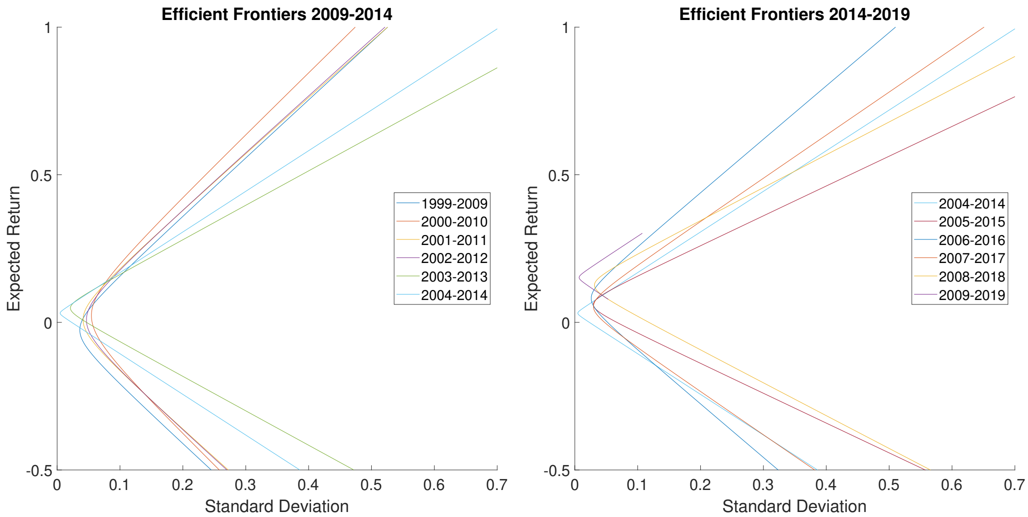
We then proceeded computing the estimated yearly returns over a rolling window of 10 years. By doing that we are trying to understand how stocks will behave based on the trend of the previous decade. Starting from these results we then build our portfolio.
Overall it appears that yearly returns tend to be positive for almost all stocks and for all period. This is quite surprising, mainly for the European stocks as their returns depend also on the volatility of currencies (for which the exchange rate reached a minimum of 0.85 and a maximum of 1.57).

	2009	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**
WalMart	0.0430	0.0095	0.0148	0.0215	0.0508	0.0535	0.0706	0.0502	0.0552	0.0953	0.0724
Coca Cola	0.0111	0.0538	0.0689	0.1017	0.1098	0.0983	0.1268	0.1347	0.1162	0.1020	0.1286
Qualcomm	0.2484	-0.0272	0.0415	0.0950	0.1204	0.0975	0.0624	0.0280	0.0909	0.1074	0.1080
Disney	-0.0764	-0.0209	0.0256	0.0605	0.1108	0.1169	0.1283	0.1610	0.1399	0.1449	0.1740
Nike	0.1043	0.1347	0.1442	0.1582	0.1879	0.1831	0.1850	0.2156	0.1758	0.1664	0.1932
Allianz	-0.0731	-0.0248	-0.0227	-0.0141	0.1062	0.0870	0.0894	0.0726	0.0543	0.0793	0.1125
Lufthansa	-0.0215	0.0046	0.0156	0.0203	0.1055	0.0529	0.0457	0.0284	-0.0395	0.0660	0.0624
VW	0.0088	0.1207	0.1875	0.1787	0.2279	0.2237	0.2139	0.1568	0.1171	0.0742	0.1419
Adidas	0.0996	0.1552	0.1709	0.1766	0.1805	0.1812	0.1033	0.1132	0.1611	0.1417	0.1876

1.3 Efficient Frontier

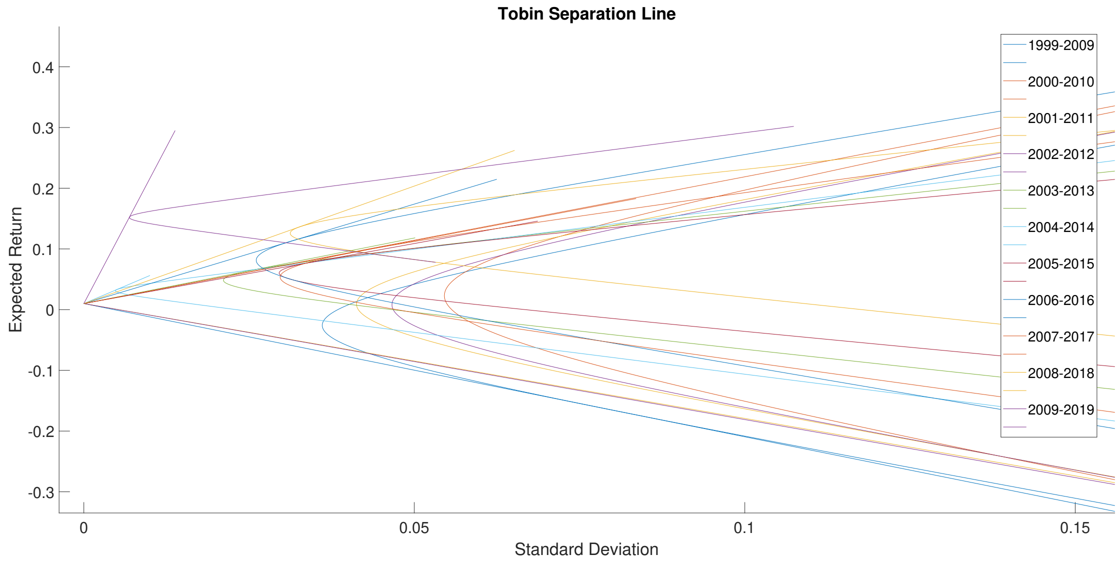
Since similar portfolios will have similar characteristics (ie return rates and standard deviation), the best strategy for an investor is to choose amongst all the possible ones those that either the maximum return given a fixed risk or the one with lowest risk for a fixed return. Such a set is what we denote as the *efficient frontier*.
The efficient frontier is a great tool when trying to gauge diversification benefits. To determine these benefits we analyse the curvature of the frontier, since it allows us to have a better understanding of the relation between the expected return rates and the risk that we would face. Different investors may have different strategies as their willingness to take risks for a higher return are different.
When including a risk-free asset, we can construct the set of efficient portfolios: adding such an assets allows us to solve the linear problem that underlies in the task of finding an optimal portfolio. Re-scaling the result without the amount invested in the risk-free asset will give us what we call the *market portfolio*. Such portfolio is also the intercept of the Tobin separation's line that is obtained by by the linear combination of the risk-free asset and the market portfolio. This set is called the *capital market line* and is the tangent to the efficient frontier. It originates from the risk-free rate and intercepts the efficient frontier in the market

portfolio, when we have our portfolio fully made by the risk-free asset and fully made by risky assets respectively.



In the plot we can see how each rolling period has different benefits. In the first one that goes from 1999 to 2009 the portfolio with the lowest risk has also a negative return; nevertheless a low increase in the risk implies a higher return rate increase than the one we can see in the 2004-2014 window, which has the lowest standard deviation for a positive return. Once again we want to highlight that based on the risk aversion of the investor, they might find more appealing completely different frontiers: one might prefer facing a higher risk for a bigger change in the expected returns, another one might prefer a lower return if this implies facing a lower risk.

In a scenario where borrowing, lending and short sales are allowed, adding an asset with a risk-free rate of 1% allows investors to have a broader choice amongst the possible portfolios: their portfolio can be rearranged in order to take away some risk by investing part of the capital in such asset.
This new scenario can be further studied by adding the Tobin separation's line to our plot. Similar conclusion can be drawn as those we had before: a steeper capital market line can be identified for the 2009-2019 window, while some others appear to have a negative slope, which can be interpreted as that it would have been more efficient to invest completely in the risk-free asset than in the market portfolio. It should be noted that we considered the same rate for borrowing and lending in the risk free asset.



1.4 Asset Allocation

Another way to try to identify market's trends is to fix a constant return and study how would the portfolio change over the time as a response to such changes. If the optimal portfolio does not need to change drastically to meet our constrain we can say that the market has been stable. Otherwise if significant changes in the market have taken place, investors have to modify their portfolios to align with the new scenario. One tool that can be used to retrieve such insights is the so called portfolio turnover, that has been computed with the following

$$turnover = \frac{\min(\text{sold, bought})}{\sum |\text{portfolio weights}|} \cdot 100$$

where sold and bought clearly indicate the units of stocks sold or bought over a time period.
It is then clear that in order to achieve a constant return it might be required to change drastically our portfolio, as it happens between 2012-2013 where the turnover is 154%, but it as likely that no major action is needed to keep return at the same level, as in the window from 2011 to 2012 where we have a turnover of 7.5%.

	2009-2010	**2010-2011**	**2011-2012**	**2012-2013**	**2013-2014**	**2014-2015**	**2015-2016**	**2016-2017**	**2017-2018**	**2018-2019**
Turnover	35.2898	30.0759	7.5230	154.9565	143.0815	41.6187	34.5924	21.0013	22.4840	74.3947

Moreover studying the portfolio turnover enables us to have a first glance of what the real return might have been after each year. Recall that our portfolio are built based on the data of the previous decade, which does in no way imply that it will keep the same return. As we can see we miss our target return almost every period. This perfectly embodies one of the biggest issue when choosing a portfolio: it will always trail behind market trends. Because of this we mainly fail to hit our targeted return, which does not necessary imply that we are on the losing side: bad performing portfolios are as likely as over performing portfolios as we can see by the study of the ex-post returns. Even if we hit negative returns of -0.68, our portfolio managed to reach a staggering return of 11.33.

	2009-2010	**2010-2011**	**2011-2012**	**2012-2013**	**2013-2014**	**2014-2015**	**2015-2016**	**2016-2017**	**2017-2018**	**2018-2019**
Ex-Post Return	-0.0937	0.5856	0.1258	-0.2624	0.4468	11.3361	-0.6838	-0.2826	0.4798	0.0884

Both of this results are in line with those we got from the turnover analysis. This is due to the fact that in both cases we want to adjust our portfolio to perform with a fixed target either being higher or lower than the one we got in the previous period.

1.5 CAPM comparison

When analysing portfolio's performance, it is helpful to understand how the empirical results behave compared to the theoretical results. In order to do so, we want to compare how stocks in the portfolio are performing compared to the corresponding market. In our case we will compare the American stocks in our portfolio with the S&P500 market index and the European ones with the DAX market index, since we expect them to be the most suitable to describe trends for our portfolio. We then proceed to compute the β for each stock against the corresponding market's index. At this point in has been quite a challenge to understand how to compare our portfolio against the two market's indices. We chose to both study each "*sub-portfolio*" against the corresponding index and to study the portfolio as whole using a weighted market index, based on how many units of American or European stocks we have in our dynamic portfolio. We acknowledge that these choices might not be the most suitable, as taking sub-portfolios of optimal portfolios does not imply that they keep being optimal, as well as having a weighted market portfolio using our stock weights does necessarily imply that it is optimal. This said, having to deal with a dynamic portfolio, which lead to have a dynamic portfolio β, these two options seemed the more reasonable and suitable for our study.
The results we see are quite insightful: even if there are periods where the CAPM prediction is (almost) met, it prevails the latter case. This is not unexpected: as we highlighted previously our predictions are based on past results so we are not entirely surprised. We also know from empirical results that sometimes it is even

more advantageous to bet against beta as presented in the work by Pedersen & Frazzini.
We can also see that the study of Jensen's alphas support this reasoning. As we get both negative and positive values it is once more clear that having a portfolio that has a target constant return might be as likely winning as losing strategy.

	2010	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**
α	-0.0902	1.3050	0.8856	-0.2629	0.4427	11.3243	-0.7017	-0.3014	0.4474	-0.1808

As a conclusion of the portfolio analysis we compute the Treynor-Mazuy measure. For this part we used the returns for each year of our portfolio and compared it to the weighted portfolio of the two market indices.

For the portfolio to have a negative timing means that it fails to react quickly

Timing ability	**Corresponding p-value**:
-6.7791	0.0031

enough to variations in market's trends. This also explains why we barely have a portfolio with constant return. It is important to note that, even if the portfolio fails to have a good timing it manages to not always being losing when compared to the market as we have seen. It should be noted that this measure has been implemented using as a term of comparison the mean return of the weighted combination of the two market indices. As previously stated, we find this to be the most immediate and natural way of dealing with the situation when our portfolio is composed by stocks from different markets, but we acknowledge that that probably is not the optimal solution.

2 Bonds

2.1 Choice and Analysis

In approaching this section we tried to collect data that could fit the kind of calculation required when working with bonds. We then found a database that provided us with the required information on the real bonds (BorsaItaliana.it). The bonds we analyzed are (all prices quoted in euro):

1. Siemens Fin Tf 2,875% Mz28 Eur. A corporate bond from ExtraMOT market.
2. Mediobanca Mb20 Tv Cap Floor Lg23 Eur. An Italian bank bond issued by MOT market.
3. Bund Tf 1,25% Ag48 Eur. A government bond issued by MOT market.
4. Daim Int Fin Tf 0,25% Mg22 Eur
5. Austria Tf 1,5% Nv86 Eur. A government bond issued by the Austrian Republic.

For a better understanding:

	Siemens	**Mediobanca**	**Bund**	**Daim**	**Austria**
Price	118.0000	103.8837	127.8749	100.3010	125.0000
Coupon Rate	0.0288	0.0300	0.0125	0.0250	0.0150

We then calculated the yield curve, duration and convexity for each bond in the portfolio. We got the following results:

	Yield	**Duration**	**Convexity**
Siemens	-0.0005	5.6460	36.6
Mediobanca	0.0029	1.4077	2.7
Bund	0.0017	23.1144	594.1
Daim	0.0154	0.3153	0.3
Austria	0.0098	44.2196	2477.2

The *yield-to-maturity* is the total return on a bond if it is held until the maturity date. The yield curve does not seem to be very high, this could be due to the perceived stability of the bonds. Generally high yield bonds, also called *junk-bonds*, have a much higher default risk, which is offset by more impressive yields than the ones in our table. The bonds in our portfolio represent relatively stable entities such as the state of Germany and international companies which have low default

risk.
Duration measures bond's sensitivity to changes in the interest rate. The table shows the modified duration in years, which is a measure of the price changes on a given bond if the interest rate changes by 1%. When investing in bonds one has to be aware of two risk factors affecting the value of investment: *credit risk* (default) and *interest rate risk* (fluctuations in the interest rate). Duration is a measure of the second risk factor and is therefore very useful when trying to determine the risk-profile of an investment in bonds.
Lastly we calculated *convexity* which is very insightful when paired with duration. Convexity is a measure of the *acceleration* of *price change* on a bond as the interest rate changes. One thing that caught our attention in this table, is that the convexity and duration is much greater on the Austrian and German state bonds (especially the Austrian). This could be due to the difference in maturity date when these two bonds are compared to the rest of the portfolio. For example the Austrian bond is set to mature in 2086, the German bond in 2048, while other maturities are in 2022, 2023 and 2028. Duration and convexity are positively correlated with maturity. So the longer the maturity, the greater the convexity/price sensitivity to yield changes.

2.2 Portfolio

In this part we calculated the duration and convexity of a portfolio of the 5 bonds before, if we invested a total of €500.000 in the portfolio. We calculated the modified duration and yearly convexity as the weighted average of the individual bond duration and convexities comprising the portfolio. We see that the weighted modified duration and convexity for the portfolio of bonds are given as 14.94 and 622.16.
Interest rates and bond prices are inverse, this means that when interest rises, bond prices decrease. So when the interest rate changes, the yield curve will shift, presenting a yield curve risk to investors. The results in this part indicate that a 1% rise in the interest rate will prompt a decrease in portfolio price of 14.94 with a rate of acceleration of 622.16. Typically a higher coupon rate or yield on the bonds will generate lower convexity. This is due to the fact that when bonds are having high yields, interest rates will have to rise more drastically in order to expose investors to risk.

2.3 Potential Decline

Here we analyzed the price changes, given a change in the interest rate of 150 basis points. The original portfolio price was €500.000 and the yield curve shifted up by 1.5%. As mentioned before, when the yield curve shifts upwards, prices should fall

because of the inverse relationship. Our approach in this section was to do some price approximations based on duration and convexity and then compare these to the true prices calculated by using the new yield curve after the upward shift.
The first order price approximation based on duration was 387.950 - or a decline in portfolio price by 22.41%. The second order price approximation based on convexity was 422.940 - or a decline in portfolio price by 15.41%.
We then calculated the true new price which was 394.110, which indicates that the approximation based on duration was the most accurate of the two. The importance of convexity in the approximations increases as the magnitude of the yield curve shift increases. This makes sense, as convexity expresses the acceleration rate of price changes given shifts in the yield curve.